Gravity and Ground Plane Geometry in Perspective Images

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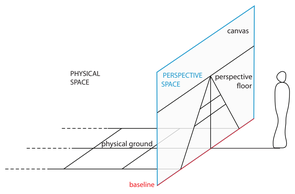
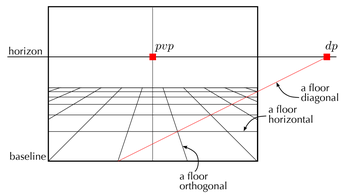
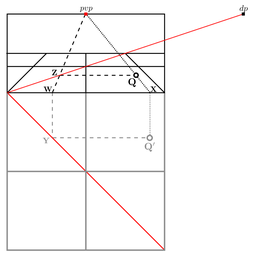
Renaissance artists noticed that placing objects on a visible ground plane anchors them stably, making it easy to perceive their depth. Subsequently, they developed methods for geometric calculation of perspective by drawing construction lines defining the ground plane. Thus, the artist constructs the geometry of pictorial space, based on the station point and the view direction, then places objects in it. This practice created a scale for placing and sizing objects on a minimally patterned ground plane.

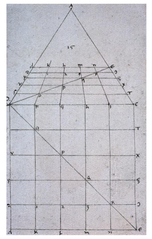


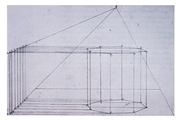


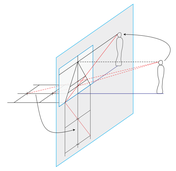
In Renaissance scene paintings feet are usually visible; our pilot experiment shows that the feet provide the viewer with an accurate perception of relative depth, configuring the scene elements into clusters. The underlying cause lies in the structure of vision, which privileges the horizontal ground and downward gravity.

Geometric Constructions for the Perspective Ground









On the left – The diagram shows the line intersections method for transferring an arbitrary floor point between the physical floor and the perspective floor, which are on either side of the baseline.

On the physical floor
- a vertical through $\mathbf{Q}'$ intersects the baseline at $\mathbf{X}$,
- a horizontal through $\mathbf{Q}'$ intersects Piero's physical diagonal at $\mathbf{Y}$, and
- a vertical through $\mathbf{Y}$ intersects the baseline at $\mathbf{W}$.

On the perspective floor the three equivalent lines are used
- a orthogonal joins $\mathbf{X}$ to the principal vanishing point, *pvp*,
- a orthogonal joins $\mathbf{W}$ to the *pvp*, intersecting Piero's perspective diagonal at $\mathbf{Z}$, and
- a horizontal through $\mathbf{Z}$ intersects the orthogonal through $\mathbf{X}$ at $\mathbf{Q}$.

$\mathbf{Q}$ is the projection of the physical floor point $\mathbf{Q}'$ to the perspective tiled floor on the picture.

In the middle – Original diagram drawn by Piero della Francesca illustrates the diagonal construction to depict a square floor of square tiles in perspective.

Top right – Original diagram also by Piero, illustrating how to draw an octagonal volume in perspective—its base, drawn in perspective on the ground, is elevated using an orthogonal plane at the extremity of the perspective floor.

Bottom right – Diagram explaining the two 90° rotations. One fold around the central vertical of the canvas and another fold around the baseline, bring the 3D world around to the image plane.

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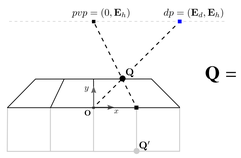
Algebraic Formalism

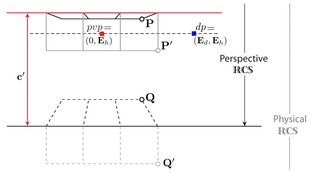
Formalizing geometric intuition, matrix algebra provides easy computation. It demonstrates the sufficiency of the reference with the ground plane—the attachment point $\mathbf{Q}$ that differentiates physical locations collocated on the image $\mathbf{P}$s—and its interaction with height $\mathbf{c}'$—a vertical distance above a relative depth defined by the point of contact with the ground. These 3D matrices are really 2D projective geometry transformations.



$$\mathbf{Q} = \begin{pmatrix} -\mathbf{E}_d & 0 & 0 \\ 0 & \mathbf{E}_h & 0 \\ 0 & 1 & -\mathbf{E}_d \end{pmatrix} \mathbf{Q}'$$

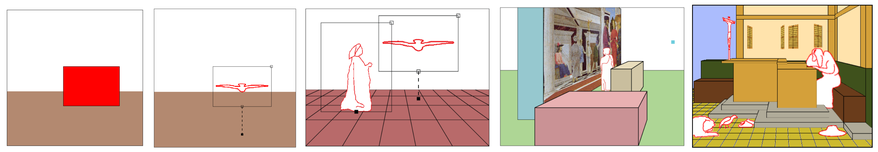
$$\mathbf{P} \sim \begin{pmatrix} -\mathbf{E}_d & 0 & 0 \\ 0 & \mathbf{E}_h & -\mathbf{c}'\mathbf{E}_d \\ 0 & 1 & -\mathbf{E}_d \end{pmatrix} \mathbf{Q}'$$

$$\mathbf{P} \sim \begin{pmatrix} \mathbf{E}_h & 0 & 0 \\ 0 & \mathbf{E}_h - \mathbf{c}' & \mathbf{c}'\mathbf{E}_h \\ 0 & 0 & \mathbf{E}_h \end{pmatrix} \mathbf{Q}$$

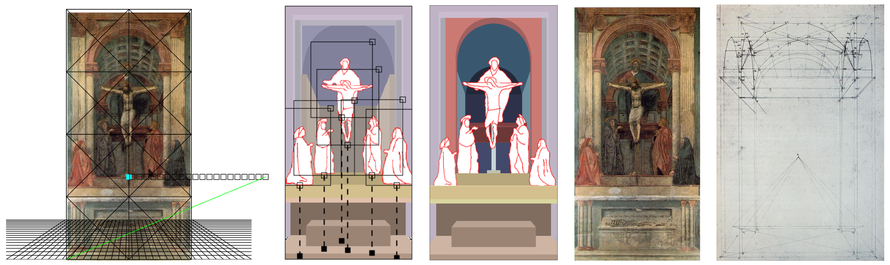


Objects & Ground: The Power of the Attachment Point

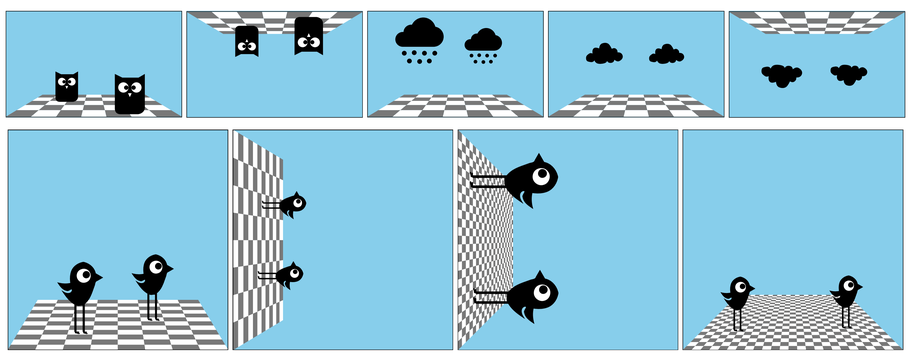
The attachment point is where an object falling under gravity would hit the ground plane. The figures below illustrate the effect of gravity and make visible some of the attachment points.



A schematic representation is built from the ground up to recreate the geometry of *The Holy Trinity* (c. 1426) by Masaccio. The image to the right is a line intersection diagram illustrating how to draw a vaulted bay in perspective, drawn by Piero della Francesca.



Creating SVG Stimuli



Pilot Experiment

Goal: Measure the precision of depth perception in perspective images.

Hypothesis: Spatial perception is influenced by two constancies

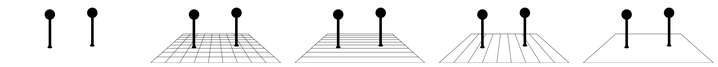
1. vertical gravity
2. ground (almost horizontal)

Ran a pilot with 12 partipants using `jspych` (https://www.jspsych.org/)

Try it online!

http://cs.colgate.edu/~efourquet/gravity/expA/expA_v2.html

The pilot experiment measures response time and accuracy of forced-choice closer/farther judgments between two objects placed in five simple scene configurations shown below.

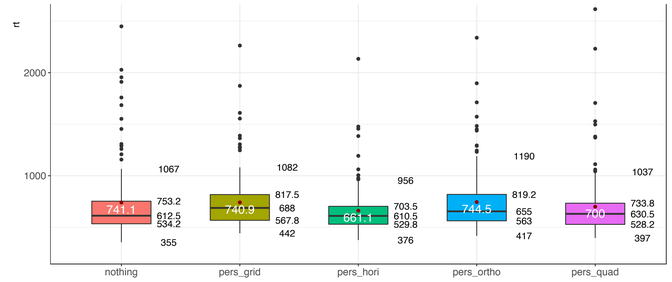


Statistically significant differences (t-test and Wilcoxon test) in response time between

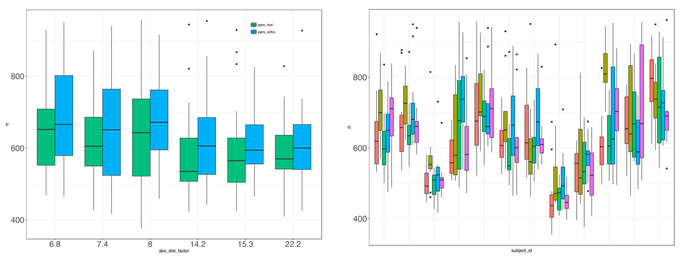
- orthogonal and horizontal pairs and
- grid and horizontal pairs.

In both cases the horizontal condition was significantly faster than the other condition.

Reaction time boxplot for the 5 conditions



Reaction times (filtered) by absolute distance and by participant



Conclusion

A prior pilot experiment showed that among the directions of gravity, participants perceive depth best when the ground plane is horizontal and gravity downward, as measured by response time at 97% accuracy.

Futhermore, the pilot experiment presented above indicates that horizontal construction lines, which abstract the horizontal guides present in the tiled floors common in Renaissance paintings, improve depth perception substantially.