Introduction to Object-Oriented Programming

Binary Search Trees

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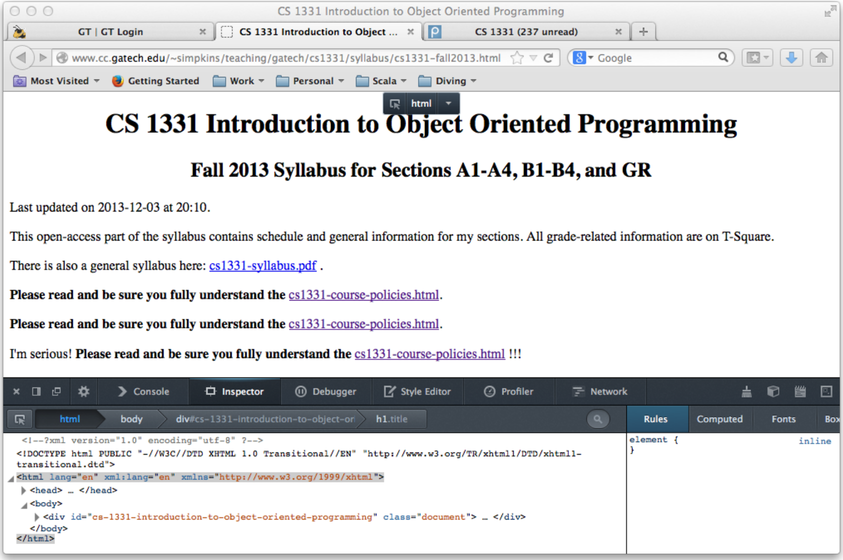
Trees are everywhere.

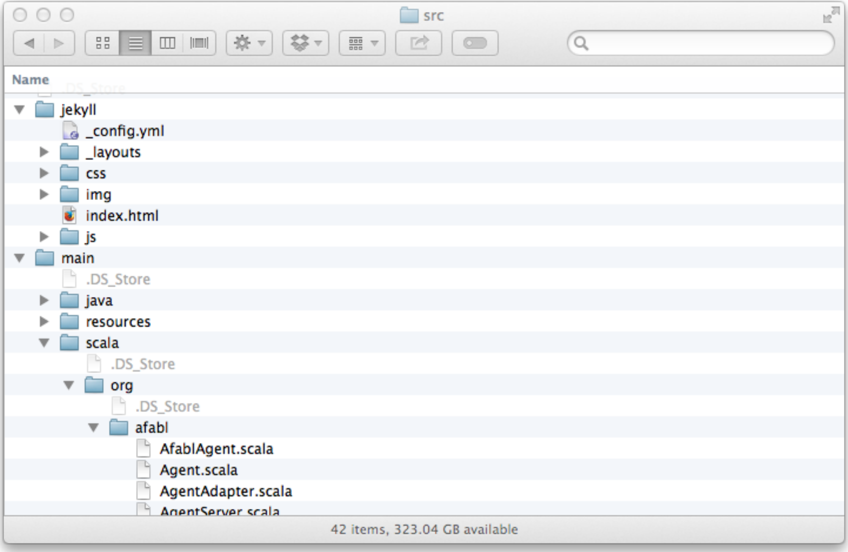

[1]

[1] Source: http://commons.wikimedia.org/wiki/File:Winnersh_Meadows_Trees.jpg

They’re in our web browsers.



They're in our file systems.



They're even in pop culture.

[2]

[2]Source:
http://userserve-ak.last.fm/serve/500/44019065/Neon+Trees.png

But they're not in Kansas.


[3]

[3] Source:
http://en.wikipedia.org/wiki/File:Wabaunsee_County_View.JPG

Binary Tree Nodes

The nodes of a binary tree have

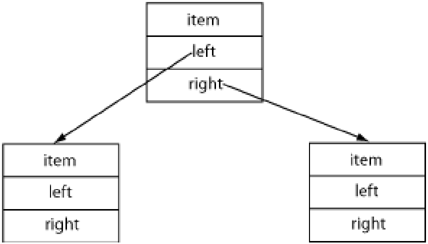
- a data item,
- a link to a left node, and
- a link to a right node.

```
private class Node<E> {
    E item;
    Node<E> left;
    Node<E> right;

    Node(E item, Node<E> left, Node<E> right) {
        this.item = item;
        this.left = left;
        this.right = right;
    }
}
```

Just as in the other linked data structures we've studied, binary trees are recursive.

Binary Tree Structure

- Every tree has a distinguished *root node* with no parent.
  - All other nodes have exactly one parent.
- Nodes which have no children are called *leaf nodes*.
- Nodes which have children are called *interior nodes*.
- Every node has 0, 1, or 2 children.
- Every node can be reached by a unique path from the root node.



Binary Search Trees

A binary search tree (BST) encodes the binary search algorithm into its structure. The BST property: for any node,

- all the elements in the node's left subtree are less than the node's data item, and
- all the elements in the node's right subtree are equal to or greater than the node's data item.

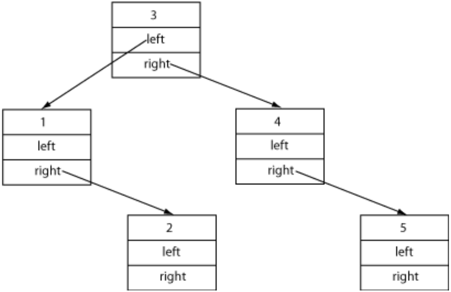
A BST is distinguished by this property, but it's ADT is just like the others we've seen: add elements, find element's in the tree, and iterate over the elements in a tree.

Maintaining The BST Property

To add a new value to binary tree and maintain the BST property, we

- insert new nodes for data items into the left subtree of a node if the new item is less than the node's item, or
- the right subtree otherwise.

Every new item creates a leaf node, which can later become an interior node after additional items have been added. Here's the structure of a BST after adding the sequence of numbers 3, 4, 1, 5, 2:

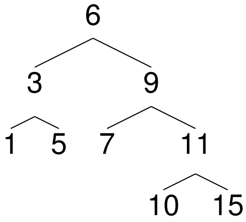


Adding Elements to a BST

```
public class BinarySearchTree<E extends Comparable<? super E>>
        implements Iterable<E> {

    protected class Node<E> { ... }

    protected Node<E> root;

    public void add(E item) {
        root = insert(item, root);
    }
    protected Node<E> insert(E newItem, Node<E> node) {
        if (node == null) {
            return new Node<E>(newItem, null, null);
        } else if (newItem.compareTo(node.item) < 0) {
            node.left = insert(newItem, node.left);
            return node;
        } else {
            node.right = insert(newItem, node.right);
            return node;
        }
    }
```

Exercise: Insertion Locations

Given the following tree that conforms to the binary search tree property:



Where would 2, 4, 8, and 16 be inserted in the tree?

Traversing a Binary Tree

There are three primary ways to traverse a binary tree:
Pre-order:

- Process node's item.
- Process left subtree.
- Process right subtree.

In-order:

- Process left subtree.
- Process node's item.
- Process right subtree.

Post-order:

- Process left subtree.
- Process right subtree.
- Process node's item.

Simple In-Order Traversal

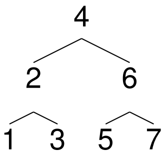
Traversal code follows the recursive structure of the tree:

```
public void printInOrder() {
    printInOrder(root);
}

private void printInOrder(Node<E> node) {
    if (node != null) {
        printInOrder(node.left);
        System.out.print(node.item + " ");
        printInOrder(node.right);
    }
}
```

The code above prints the elements in ascending order. Let's add a `printDescending()` method to BinarySearchTree.java.

Exercise: Traversal Orders

Given the following tree:



If we proecssed each element by printing it, in what order would the elements be printed

- For a pre-order traversal:
- For an in-order traversal:
- For a post-order traversal:

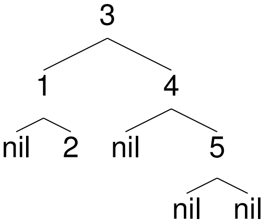
The Path to an Item

To find a path to an item in a BST:

- set the path to the empy list
- set the root node as the currentNode
- until we find the node containing the item or exhaust the BST:
  - if currentNode contains the item, add it to the path and return it
  - else if query item is less than the item in currentNode, add currentNode to path and set the left child as the new currentNode
  - else add add currentNode to path and set the right child as the new currentNode
- if the item wasn't found, set the path to the empty list

Path Examples

Adding the elements `[3, 4, 1, 5, 2]` to a BST would result in the following structure:



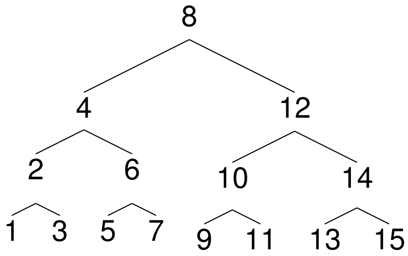
and the paths to each element in the tree would be:

- Path to 1: [3, 1]
- Path to 2: [3, 1, 2]
- Path to 3: [3]
- Path to 4: [3, 4]
- Path to 5: [3, 4, 5]

See `public List<E> path(E queryItem)` in BinarySearchTree.java for the code.

Exercise: Paths

Given the following tree:



- What's the path to 1?
- Whats the path to 11?

Recursively Building a Result: `inOrderList()`

We can use the recursive accumulator idiom to collect the elements of the tree in an in-order traversal:

```
    public List<E> toList() {
        return inOrderList(root, new ArrayList<E>());
    }

    private List<E> inOrderList(Node<E> node, List<E> accum) {
        if (null == node) {
            return accum;
        } else {
            inOrderList(node.left, accum);
            accum.add(node.item);
            inOrderList(node.right, accum);
        }
        return accum;
    }
```

Again, the code follows the recursive structure of the tree.

Imperative traveral: `inOrderImperative()`

Contrast the previous code for getting an in-order list of BST elements with an imperative version:

```
public List<E> inOrderImperative() {
    Node<E> curNode = root;
    Stack<Node<E>> fringe = new LinkedStack<>();
    List<E> accum = new ArrayList<E>();
    while ((curNode != null) || !fringe.isEmpty()) {
        while (curNode != null) {
            fringe.push(curNode);
            curNode = curNode.left;
        }
        curNode = fringe.pop();
        accum.add(curNode.item);
        curNode = curNode.right;
    }
    return accum;
}
```

We need extra bookkeeping variables to keep track of where we are in the tree so we can back-track. See BinarySearchTree.java for comments explaining the algorithm.

Iterators

Iterators can free clients from having to implement traversal algorithms. We can even plug our data structures into Java's for-each loop by implementing `java.lang.Iterable`:

```
public interface Iterable<T> {
    java.util.Iterator<T> iterator();
}
```

As a reminder, `java.util.Iterator`:

```
public interface Iterator<E> {
    boolean hasNext();

    E next();

    void remove();
}
```

Stateful In-Order Tree Traversal

In the traversal examples we saw earlier the traversal order was effected by the method call stack. A stateful iterator is much more challenging because:

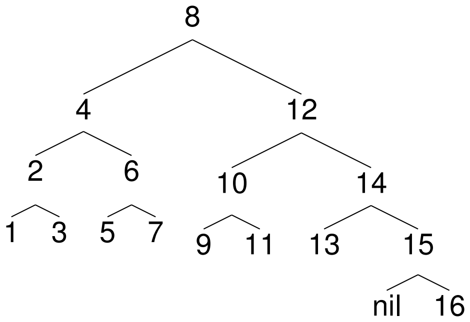
- The iterator must remember where it is in the tree
- The iterator must be able to back-track to parent nodes after processing child branches

The essential implementation idea is to use a stack to store nodes for back-tracking. Traditionally (at least in AI), this “to-do list” stack is called the *fringe*.

Let’s look at BinarySearchTree.java again to see how we implement a stateful in-order iterator.

Analysis of BSTs

What is the Big-O of finding an element in a BST?



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- Height of tree is proportional to log n
- 16-element tree has height 4

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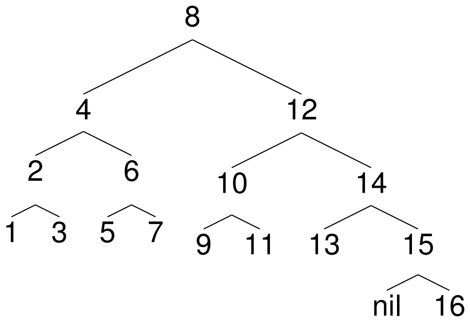

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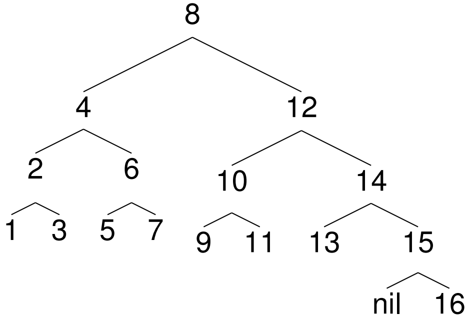

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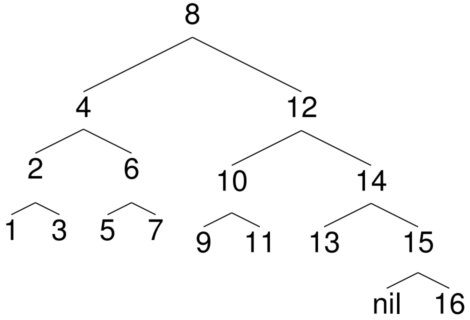

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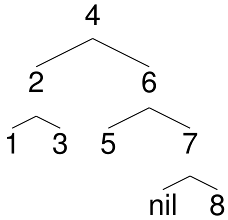
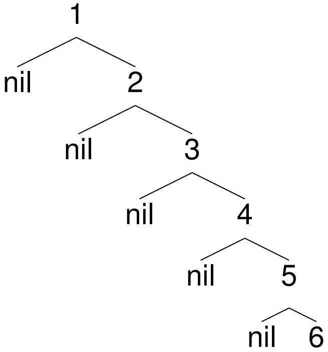
Height of BST Proportional to log n



8-element tree has height 3

End of course cliff hanger ...

What if we add elements to a BST in order?



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- In your algorithms and data structures course you'll learn how to maintain a balanced binary search tree.

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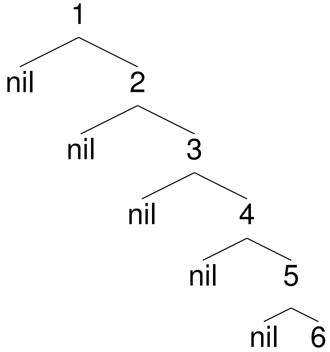

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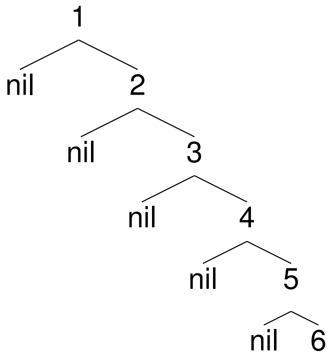

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