Faster Addition

Carry Look-Ahead
One Bit Carry Computation

\[ \text{Cout} = (A \text{ or } B \text{ and } \text{Cin}) \text{ or } (A \text{ and } B) \]
One Bit Carry

• (A or B) propagates carry-in to carry out
  • Let (A or B) = p

• (A and B) generates a carry-out, regardless of a carry-in
  • Let (A and B) = g

• Cout = g + (p \cdot Cin)  (+ means or, \cdot means and )
Four Bit Carry Computation

\[ C_1 = g_0 + p_0 \cdot C_0 \quad g_0 = A_0 \cdot B_0 \quad p_0 = A_0 + B_0 \]

\[ C_2 = g_1 + p_1 \cdot C_1 \quad g_1 = A_1 \cdot B_1 \quad p_1 = A_1 + B_1 \]
\[ = g_1 + p_1 \cdot (g_0 + p_0 \cdot C_0) \]
\[ = g_1 + p_1 \cdot g_0 + p_1 \cdot p_0 \cdot C_0 \]

\[ C_3 = g_2 + p_2 \cdot C_2 \quad g_2 = A_2 \cdot B_2 \quad p_2 = A_2 + B_2 \]
\[ = g_2 + p_2 \cdot (g_1 + p_1 \cdot g_0 + p_1 \cdot p_0 \cdot C_0) \]
\[ = g_2 + p_2 \cdot g_1 + p_2 \cdot p_1 \cdot g_0 + p_2 \cdot p_1 \cdot p_0 \cdot C_0 \]

\[ C_4 = g_3 + p_3 \cdot C_3 \quad g_3 = A_3 \cdot B_3 \quad p_3 = A_3 + B_3 \]
\[ = g_3 + p_3 \cdot g_2 + p_3 \cdot p_2 \cdot g_1 + p_3 \cdot p_2 \cdot p_1 \cdot g_0 + p_3 \cdot p_2 \cdot p_1 \cdot p_0 \cdot C_0 \]
Carry Look-ahead

C_1 = g_0 + p_0 \cdot C_0 \\
\text{g}_0 = A0 \cdot B0 \quad p_0 = A0 + B0

C_2 = g_1 + p_1 \cdot g_0 + p_1 \cdot p_0 \cdot C_0 \\
\text{g}_1 = A1 \cdot B1 \quad p_1 = A1 + B1

C_3 = g_2 + p_2 \cdot g_1 + p_2 \cdot p_1 \cdot g_0 + p_2 \cdot p_1 \cdot p_0 \cdot C_0 \\
\text{g}_2 = A2 \cdot B2 \quad p_2 = A2 + B2

C_4 = g_3 + p_3 \cdot g_2 + p_3 \cdot p_2 \cdot g_1 + p_3 \cdot p_2 \cdot p_1 \cdot g_0 + p_3 \cdot p_2 \cdot p_1 \cdot p_0 \cdot C_0 \\
\text{g}_3 = A3 \cdot B3 \quad p_3 = A3 + B3
Second stage look-ahead

• In the look-ahead part of the circuit
  1. What would propagate C0 to C4?
  2. What would constitute a generate of C4?
Second stage look-ahead

• In the look-ahead part of the circuit
  1. What would propagate C0 to C4?
  2. What would constitute a generate of C4?

1. Propagate: \( P_0 = p_3 \cdot p_2 \cdot p_1 \cdot p_0 \)

2. Generate: \( G_0 = g_3 + p_3 \cdot g_2 + p_3 \cdot p_2 \cdot g_1 + p_3 \cdot p_2 \cdot p_1 \cdot g_0 \)
Second stage look-ahead for 16-bit adder

Propagate: \( P_0 = p_3 \cdot p_2 \cdot p_1 \cdot p_0 \)
Generate: \( G_0 = g_3 + p_3 \cdot g_2 + p_3 \cdot p_2 \cdot g_1 + p_3 \cdot p_2 \cdot p_1 \cdot g_0 \)

Propagate: \( P_1 = p_7 \cdot p_6 \cdot p_5 \cdot p_4 \)
Generate: \( G_1 = g_7 + p_7 \cdot g_6 + p_7 \cdot p_6 \cdot g_5 + p_7 \cdot p_6 \cdot p_5 \cdot g_4 \)

Propagate: \( P_2 = p_{11} \cdot p_{10} \cdot p_9 \cdot p_8 \)
Generate: \( G_2 = g_{11} + p_{11} \cdot g_{10} + p_{11} \cdot p_{10} \cdot g_9 + p_{11} \cdot p_{10} \cdot p_9 \cdot g_8 \)

Propagate: \( P_3 = p_{15} \cdot p_{14} \cdot p_{13} \cdot p_{12} \)
Generate: \( G_3 = g_{15} + p_{15} \cdot g_{14} + p_{15} \cdot p_{14} \cdot g_{13} + p_{15} \cdot p_{14} \cdot p_{13} \cdot g_{12} \)
Carry Look-ahead stage 2

\[ P_0 \]
\[ G_0 \]
\[ P_1 \]
\[ G_1 \]
\[ P_2 \]
\[ G_2 \]
\[ P_3 \]
\[ G_3 \]

\[ C_0 \]
\[ C_4 \]
\[ C_8 \]
\[ C_{12} \]
\[ C_{16} \]
Second level expressions

\[ C_4 = G_0 + P_0 C_0 \]
\[ C_8 = G_1 + P_1 G_0 + P_1 P_0 C_0 \]
\[ C_{12} = G_2 + P_2 G_1 + P_2 P_1 G_0 + P_2 P_1 P_0 C_0 \]
\[ C_{16} = G_3 + P_3 G_2 + P_3 P_2 G_1 + P_3 P_2 P_1 G_0 + P_3 P_2 P_1 P_0 C_0 \]
8 Bit Carry Look-Ahead

\[ C_1 = g_0 + p_0 C_0 \]

\[ C_2 = g_1 + p_1 g_0 + p_1 p_0 C_0 \]

\[ C_3 = g_2 + p_2 g_1 + p_2 p_1 g_0 + p_2 p_1 p_0 C_0 \]

\[ C_4 = g_3 + p_3 g_2 + p_3 p_2 g_1 + p_3 p_2 p_1 g_0 + p_3 p_2 p_1 p_0 C_0 \]

\[ C_5 = g_4 + p_4 g_3 + p_4 p_3 g_2 + p_4 p_3 p_2 g_1 + p_4 p_3 p_2 p_1 g_0 + p_4 p_3 p_2 p_1 p_0 C_0 \]

\[ C_6 = g_5 + p_5 g_4 + p_5 p_4 g_3 + p_5 p_4 p_3 g_2 + p_5 p_4 p_3 p_2 g_1 + p_5 p_4 p_3 p_2 p_1 g_0 + p_5 p_4 p_3 p_2 p_1 p_0 C_0 \]

\[ C_7 = g_6 + p_6 g_5 + p_6 p_5 g_4 + p_6 p_5 p_4 g_3 + p_6 p_5 p_4 p_3 g_2 + p_6 p_5 p_4 p_3 p_2 g_1 + p_6 p_5 p_4 p_3 p_2 p_1 g_0 + p_6 p_5 p_4 p_3 p_2 p_1 p_0 C_0 \]

\[ C_8 = g_7 + p_7 g_6 + p_7 p_6 g_5 + p_7 p_6 p_5 g_4 + p_7 p_6 p_5 p_4 g_3 + p_7 p_6 p_5 p_4 p_3 g_2 + p_7 p_6 p_5 p_4 p_3 p_2 g_1 + p_7 p_6 p_5 p_4 p_3 p_2 p_1 g_0 + p_7 p_6 p_5 p_4 p_3 p_2 p_1 p_0 C_0 \]
8 Bit Carry Look-Ahead – Second Stage

\[ G_0 = g_7 + p_7g_6 + p_7p_6g_5 + p_7p_6p_5g_4 + p_7p_6p_5p_4g_3 + p_7p_6p_5p_4p_3g_2 \]
\[ \quad + p_7p_6p_5p_4p_3p_2g_1 + p_7p_6p_5p_4p_3p_2p_1g_0 \]

\[ P_0 = p_7p_6p_5p_4p_3p_2p_1p_0 \]

\[ G_1 = g_{15} + p_{15}g_{14} + p_{15}p_{14}g_{13} + p_{15}p_{14}p_{13}g_{12} + p_{15}p_{14}p_{13}p_{12}g_{11} \]
\[ \quad + p_{15}p_{14}p_{13}p_{12}p_{11}g_{10} + p_{15}p_{14}p_{13}p_{12}p_{11}p_{10}g_9 + p_{15}p_{14}p_{13}p_{12}p_{11}p_{10}p_9g_8 \]

\[ P_1 = p_{15}p_{14}p_{13}p_{12}p_{11}p_{10}p_9p_8 \]

Etc.
Carry levels of logic

• 64-bit ripple carry
  • 2 levels each 1-bit adder
  • 128 levels total

• 64-bit look-ahead carry, 8-bit blocks, 8 2\textsuperscript{nd} level blocks
  • 1 level each for p, g
  • 2 level for G (one level for P)
  • 2 levels for 2\textsuperscript{nd} level carries
  • 2 levels back through first level
  • 2 levels last sum
  • 9 levels total
  • 14 times as fast!